Nonlinear Transient Analysis via Energy Minimization

Manohar P. Kamat*
Virginia Polytechnic Institute and State University, Blacksburg, Va.
and
Norman F. Knight Jr.†
NASA Langley Research Center, Hampton, Va.

Abstract

Nonlinear transient analysis of structures has been of increasing interest to engineers by virtue of their interest in minimizing human and property damage resulting from the catastrophic failure of such structures under crash or seismic conditions. Complexities of the structural configuration and its equally complex transient response in the presence of material inelasticity make finite element modeling of such structures a very natural and plausible recourse.

For solution two distinct approaches exist: 1) the vector approach and 2) the scalar approach. In the former, a mathematical model is derived on the basis of the principle of virtual work and reduces to a system of nonlinear second-order differential equations in time. In the latter approach, a potential function associated with the energy of the model is introduced, minimization of which yields the desired equilibrium configuration. In both approaches a temporal finite-difference scheme is utilized to effectively eliminate time as a variable. In the scalar approach the problem is then reduced to a well known problem in mathematical programming; namely, the unconstrained minimization of a nonlinear function of several variables.

The scalar approach, although used by previous investigators for nonlinear analysis, was, with the exception of Ref. 1, restricted to static conditions. The algorithm of Ref. 1 had difficulties in converging to correct solutions because of inherent element formulation deficiencies and the use of highly expensive and inefficient finite-difference operations for gradients, besides being restricted to stringer and frame element models. As a result, no meaningful results using energy minimization were obtained. The present formulation overcomes such limitations using analytically derived gradients, an extension of the element library coupled with consistent element formulations, and the best current variable metric update formula for use in unconstrained minimization.[2]

Contents

Brief Formulation Details

The displacement-time relation for each generalized nodal displacement of the finite element model is assumed to be of the form

$$q_{ei}=\beta(\Delta t)^2\ddot{q}_{ei}+(½-\beta)(\Delta t)^2\ddot{q}_{oi}+(\Delta t)\dot{q}_{oi}+q_{oi} \qquad (1)$$

$$\dot{q}_{ei}=\gamma(\Delta t)\ddot{q}_{ei}+(1-\gamma)(\Delta t)\ddot{q}_{oi}+\dot{q}_{oi} \qquad (2)$$

where q_{ei} is the ith generalized nodal displacement at the end of the time step, β and γ are constants, and q_{oi}, $\dot{q}_{oi}$ and $\ddot{q}_{oi}$ are the ith generalized nodal displacement, velocity, and acceleration at the beginning of the time step. The equations of equilibrium for an N-degree-of-freedom system with lumped masses

$$M_i\ddot{q}_{ei}-F_i+\frac{\partial U}{\partial q_{ei}}=0 \qquad i=1,2,...N \qquad (3)$$

can be shown to be the necessary conditions for the functional

$$S=\sum_{i=1}^{N}\left\{\left[\frac{1}{2\beta(\Delta t)^2}q_{ei}^2-\left(\frac{1}{\beta(\Delta t)}q_{oi}+\frac{1}{\beta(\Delta t)}\dot{q}_{oi}+\left(\frac{1}{2\beta}-1\right)\ddot{q}_{oi}\right)q_{ei}\right]M_i-F_iq_{ei}\right\}+U+C \qquad (4)$$

to be stationary. U is the strain energy and C is a constant. The size of the time step Δt is automatically controlled so that the error at half time based on interpolated configuration is less than a prescribed change in total energy.

For minimization of S, use is made of the well-known BFGS algorithm[2] that dispenses with the exact line searches while using an update formula which, in the case of quadratic functions, guarantees convergence of the eigenvalues of the approximating matrix to the inverse hessian. The required gradient of S is evaluated analytically,[3] with the term $\partial U/\partial q_{ei}$ requiring significant computational effort since it embraces the geometric and material nonlinearities.

The complexity of the strain energy evaluation for any element is determined by its deformation and material models. The deformation model of the entire structure is synthesized from deformation states of each element. The displacement field within each element is a continuously differentiable function of the local spatial coordinates and the generalized nodal displacements and is forced to maintain interelement continuity of its essential derivatives. The local nodal displacements of each element are then related to the global displacements of the assemblage. For large, rigid-body rotations, these relations, which can be interpreted as transformations of the local to the global coordinate system, are accomplished using Euler angles that are linearly independent because the rotations are performed in a prescribed order. With the restriction of small relative rotations within the element, the corotational formulation leads to a simplification of the strain-displacement relationship on the element level while still permitting arbitrarily large rotations of the element. Through appropriate kinematic constraints, modeling of rigid links or even the simulation of contact with an impenetrable, rough plane are easily achieved.

Von Mises' yield criterion with Henckey's deformation theory, utilizing the effective strain-effective stress concept, provide a simple means of calculating the strain energy of an element that has yielded. For modeling plasticity under cyclic loading, kinematic hardening with an idealized Bauschinger effect is assumed. Specialized elements such as gaps and stays are modeled simply through an appropriate modification of the material model of the conventional elements. Because total stresses and strains are no longer linearly related, recourse must be made to numerical integration for the calculation of the total strain energy and the stress resultants.

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*Associate Professor, Dept. of Engineering Science and Mechanics. AIAA Member

†Graduate Research Assistant, George Washington University Center.

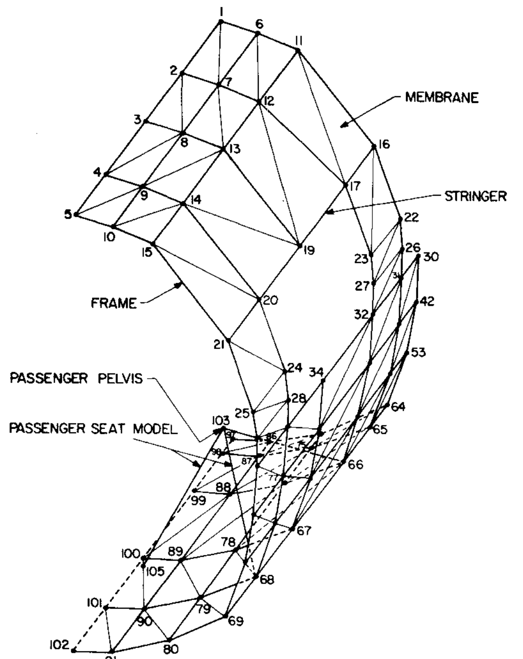

Fig. 1 Finite element model of an aircraft fuselage substructure.

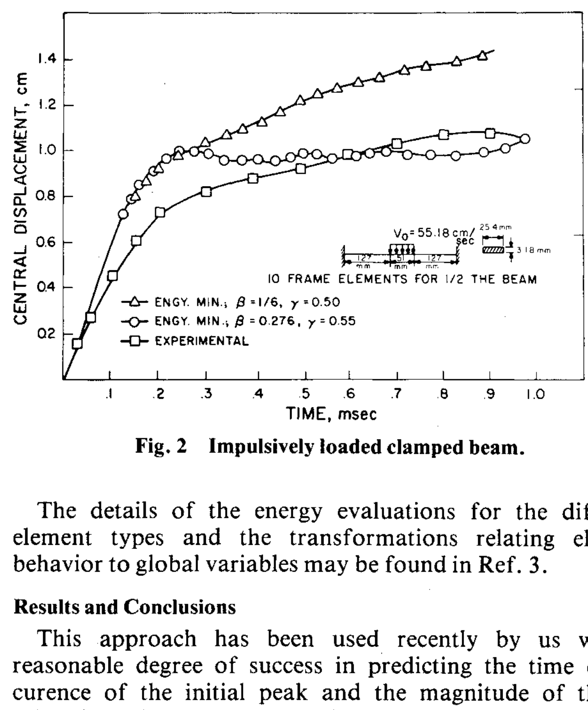

Fig. 2 Impulsively loaded clamped beam.

The details of the energy evaluations for the different element types and the transformations relating element behavior to global variables may be found in Ref. 3.

Results and Conclusions

This approach has been used recently by us with a reasonable degree of success in predicting the time of occurence of the initial peak and the magnitude of the acceleration of the occupant inside the substructure of an aircraft under a vertical drop. The model of Fig. 1 involved a total of 336 degrees-of-freedom, 105 nodes, and 209 elements (including 96 membranes, 77 frame elements, and 36 stringer elements). Reference 4 provides the model and simulation details. For the purposes of this paper two well-known but classical and rather small-scale problems for which test data are readily available are considered.

Figure 2 illustrates the case of transient response in the presence of both geometric and material nonlinearities. The impulse is large enough to cause the entire beam to respond inelastically while experiencing moderately large relative rotations. The quality of the response prediction is very much a function of β and γ, optimum values of which are very much problem dependent. The discrepancy between the analysis and test is most likely the result of the implied boundary conditions of the analytical model being different from those of the actual test.

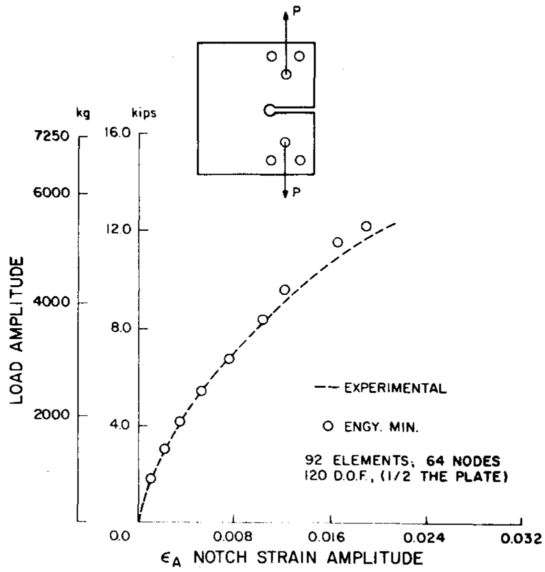

Fig. 3 The notch problem.

In Fig. 3, using constant strain triangular membrane elements, the maximum strain in the vicinity of the notch in the direction of loading is determined and compared with the experimental results. The agreement between analysis and test could perhaps be improved upon by the use of nonlinear strain displacement relationship in the corotational coordinate system.

Indeed, one may always claim that this demonstration of the effectiveness of the minimization technique as a tool for nonlinear analysis has been restricted to problems with relatively few degrees-of-freedom. For such small-scale classical problems the energy minimization technique has been shown to be at least comparable to, if not better than, the pseudoforce technique of the vector approach.[5] Extensions to large-scale problems such as a full aircraft may involve several thousands of degrees of freedom. It is our opinion that the state-of-the-art in nonlinear transient analysis in general does not appear to be at a point where such large-scale problems can be solved efficiently and with any high degree of confidence in the simulation fidelity. Likewise, the effectiveness of the present technique for response prediction of such large scale structures remains to be demonstrated. Using preconditioned conjugate gradient techniques or variable metric methods which exploit sparsity, it is believed that this will no longer be an insurmountable task.

Acknowledgment

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